

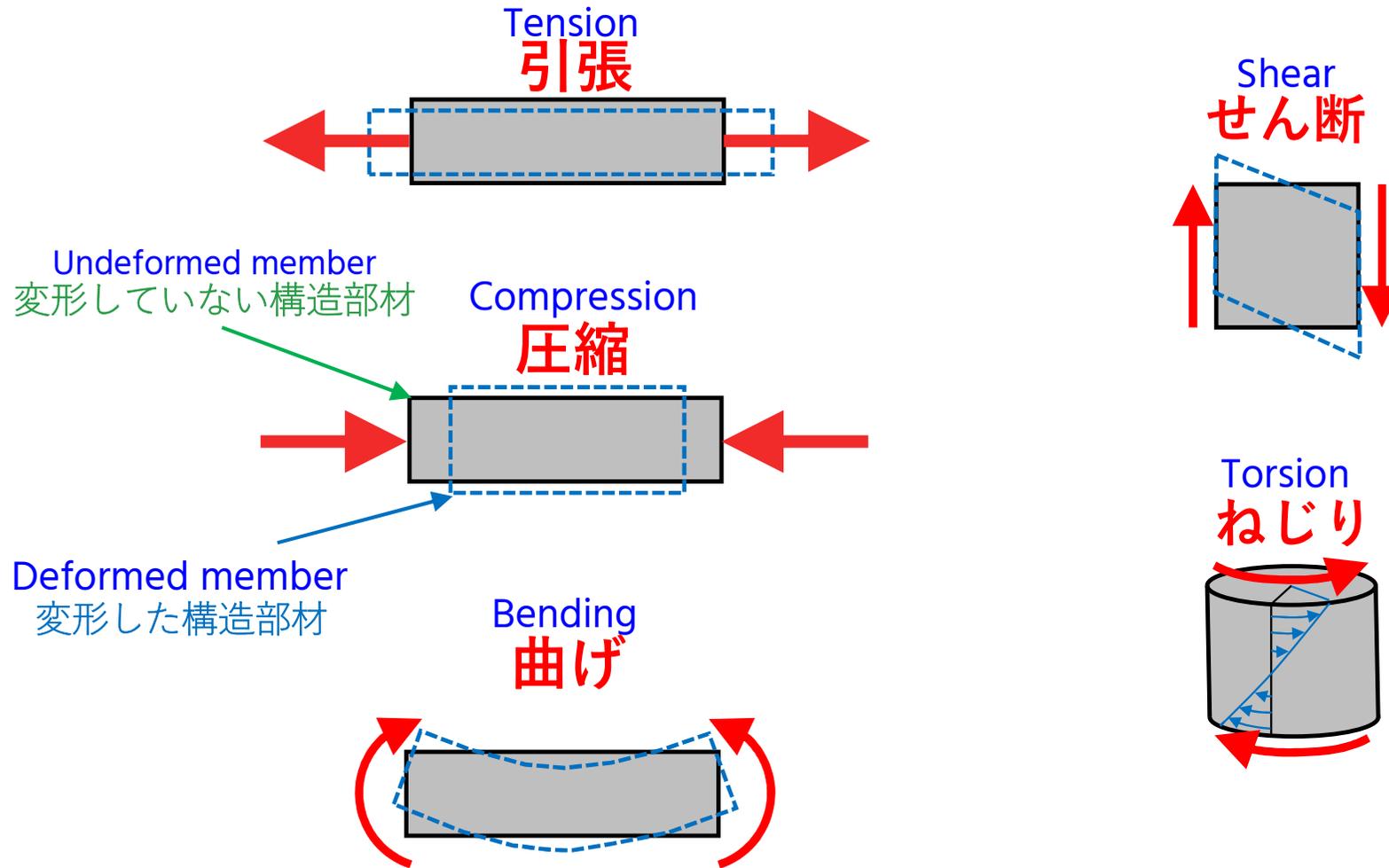
非線形コンクリート特論 (Nonlinear Behavior of Concrete and Concrete Members)

# (1) ひずみ・Strain

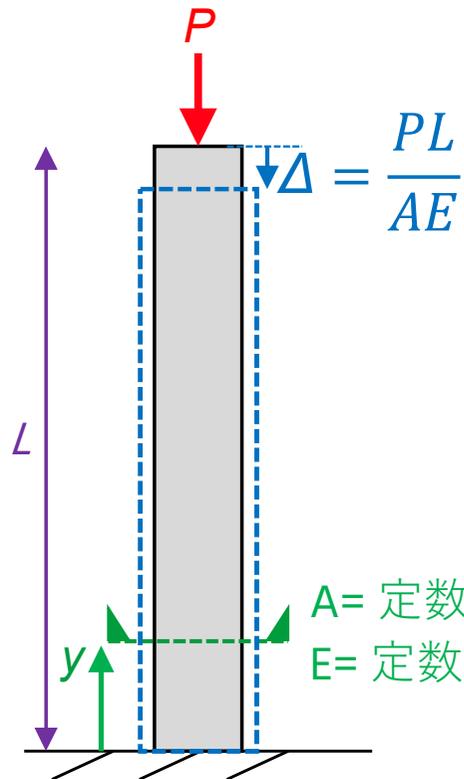
Trevor Zhiqing YEOW (ヤオ トレボー ジキン)

東京科学大学 総合研究院 多元レジリエンス研究センター ・ 特別助教

# 構造部材の応答・Member response

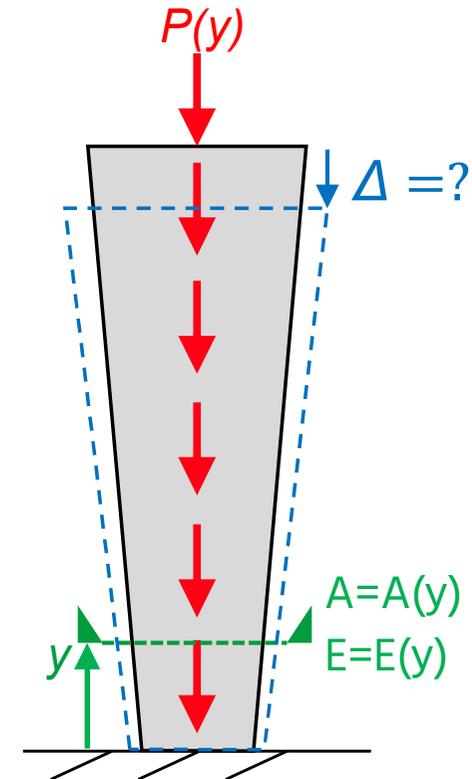


# 構造部材の応答・Member response



一定の軸力と構造部材特性

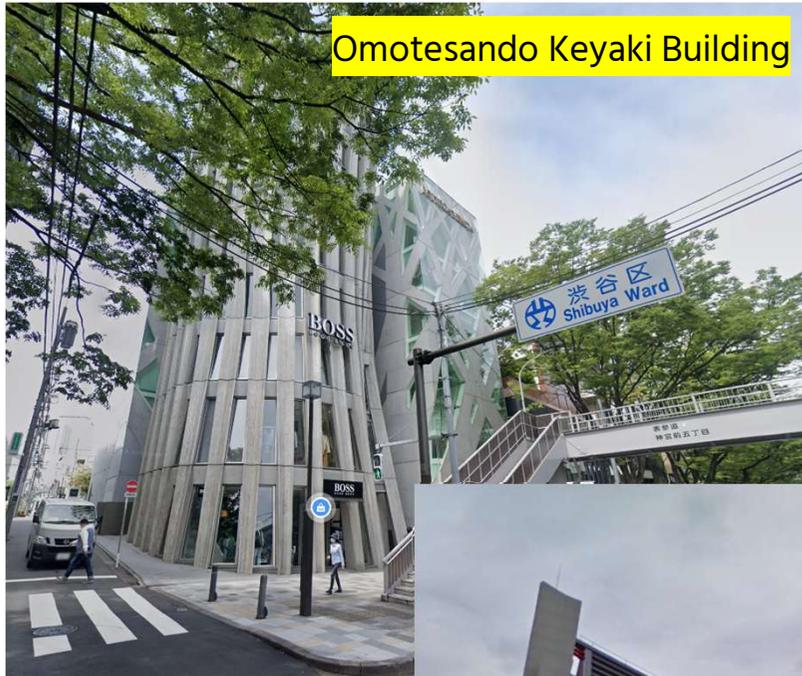
Constant axial force and member properties



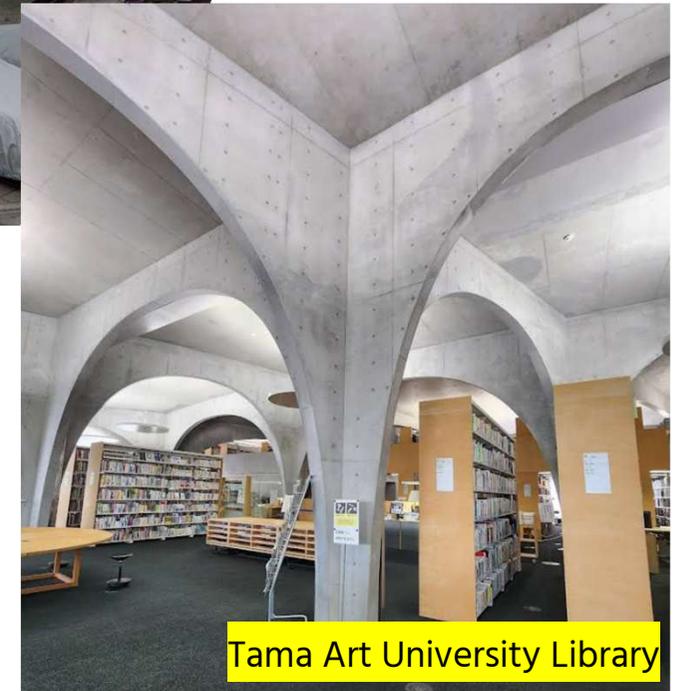
可変軸力と構造部材特性

Variable axial force and member properties

# 本物の建物・Real Buildings



Taken by Raja Vijayaraghavan,  
obtained from Google Maps



All images obtained from Google Street  
View unless otherwise stated

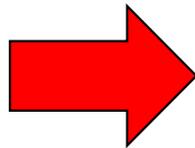
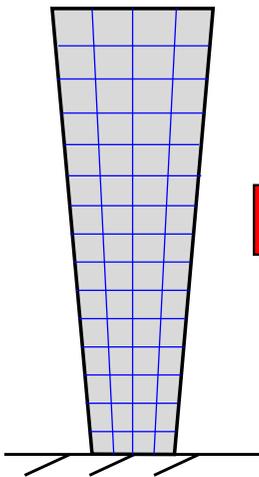
# 構造部材の応答・Member response

- 厳密解だが難しい

The exact solution may be complicated

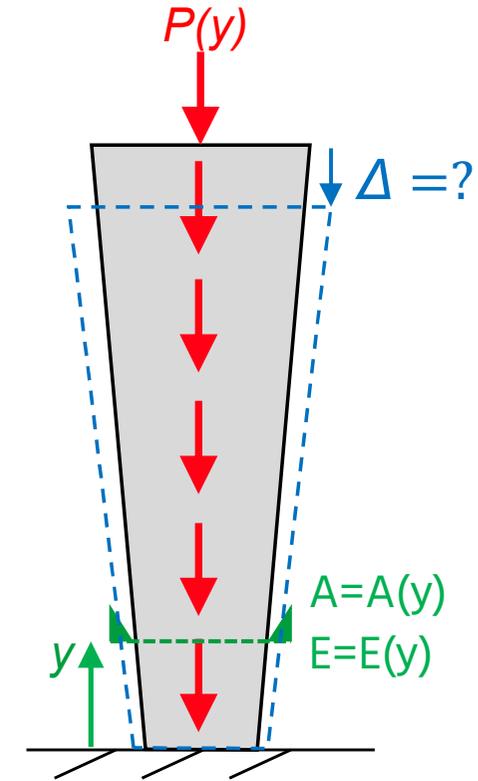
$$\Delta = \int_0^L \frac{P(y)}{A(y)E(y)} dy$$

- 数値解法      Numerical analysis  
(例：構造部材をメッシュに分解)  
(Example: discretizing member into a mesh)

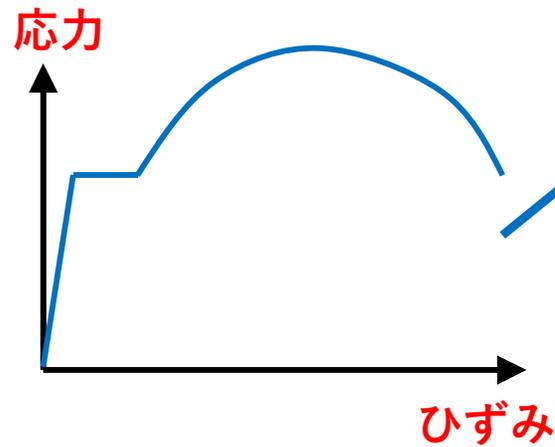


Such analysis methods will be covered in more detail later in the course

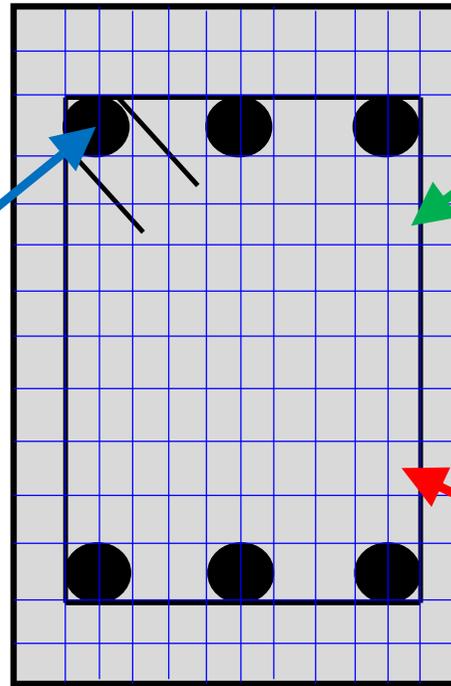
このような数値解析手法はコースの後半で詳しく説明する予定です



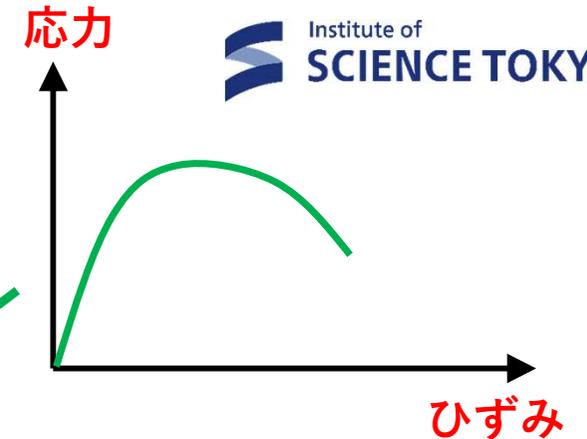
# 材料の応答・Material response



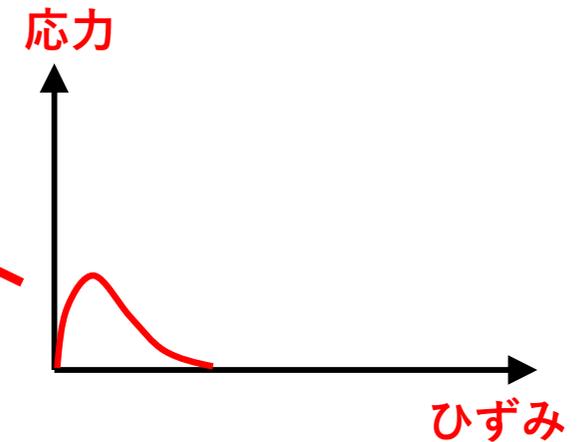
鋼鉄料  
Steel material



鉄筋コンクリート断面  
Reinforced concrete section



コンクリート (圧縮)  
Concrete (compression)



コンクリート (引張)  
Concrete (tension)

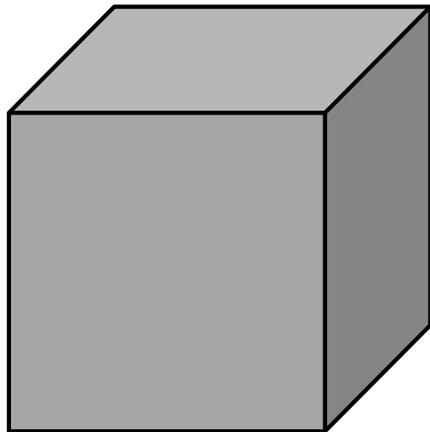
## Need to understand stress and strains in order to perform numerical simulations

数値解析手法が行えるために、  
応力・ひずみを理解することが必要である

# 等方的・異方的材料 Isotropic/Anisotropic Material

Isotropic  
等方的

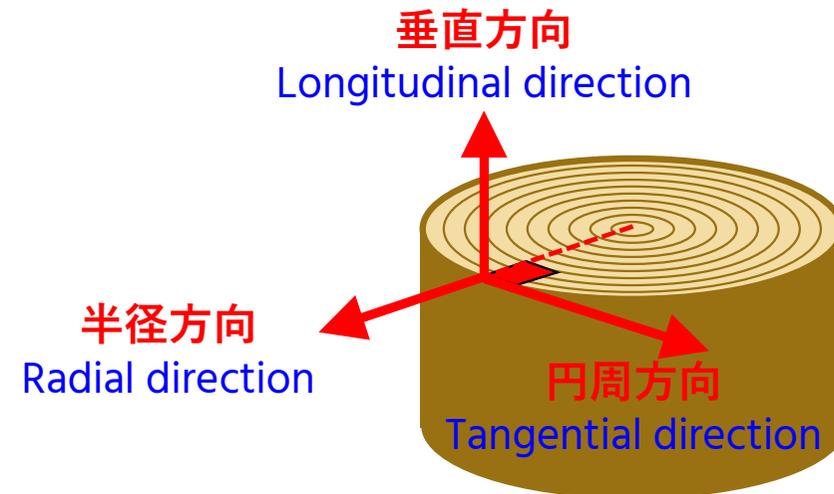
方向によらずに機械的な特性が同じ  
Mechanical properties are the same in all directions



鋼鉄材料  
Steel

Anisotropic  
異方的

方向によらずに機械的な特性が異なる  
Mechanical properties depend on direction



木材料  
Timber

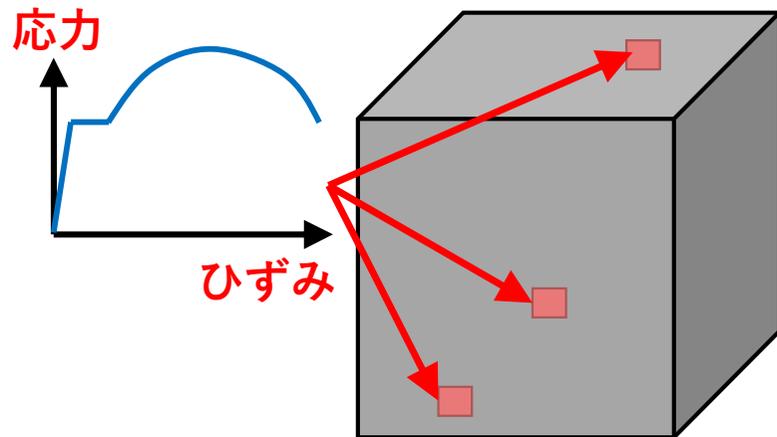
# 均質・非均質材

# Homogeneous · Heterogeneous Material

## Homogeneous 均質材

材料中のどの場所でも応力－ひずみ  
関係が同一である

Stress-strain relationship is identical throughout material



鋼鉄材料  
Steel

## Heterogeneous 非均質材

材料中の場所毎に応力－ひずみ関係が  
異なる

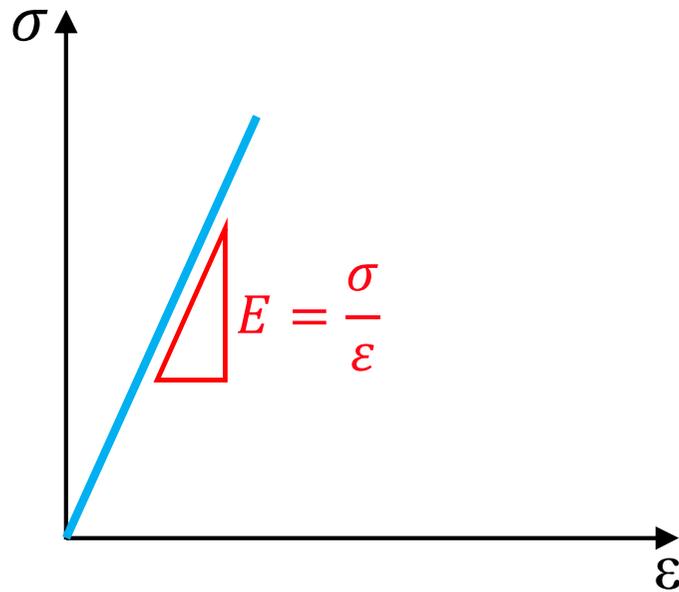
Nonuniform material composition throughout material

セメントと骨材の特性  
は異なる  
Properties of cement and  
aggregate are different



コンクリート  
Concrete

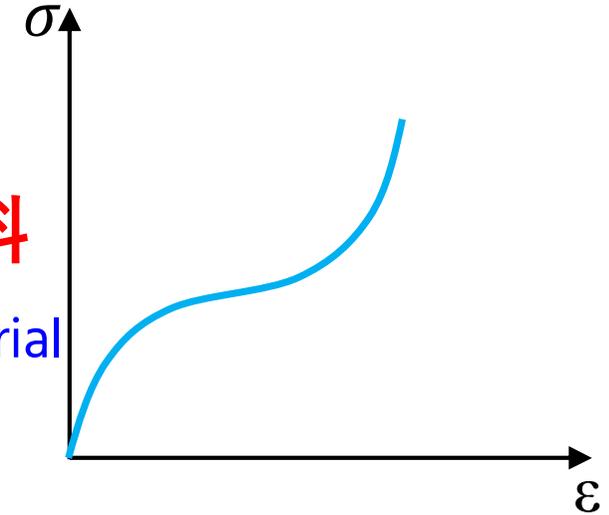
# 線型・非線型/弾性・塑性 Linear · Nonlinear/Elastic · Plastic



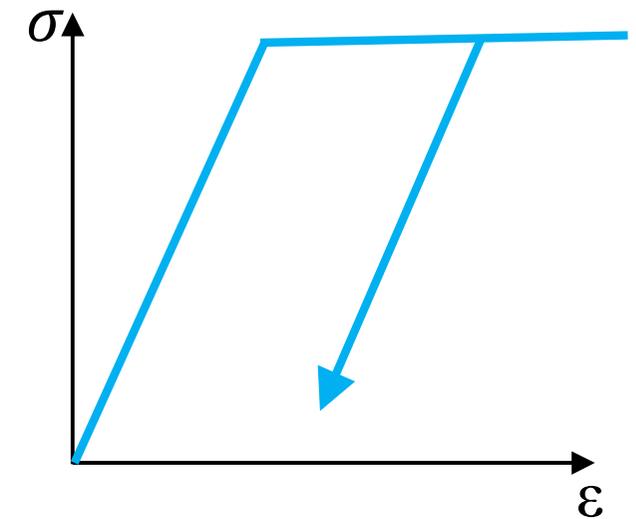
線型弾性材料

Linear elastic material

非線型材料  
Nonlinear material



塑性材料  
Plastic material

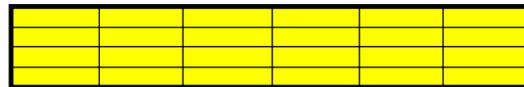
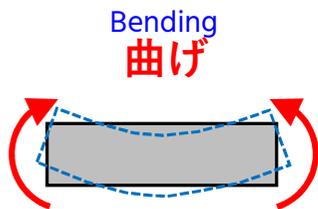
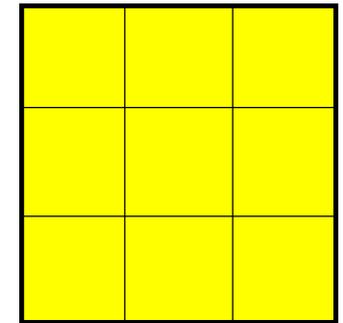
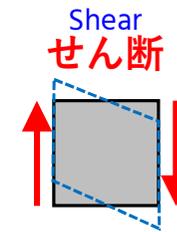
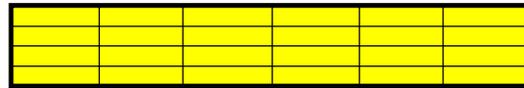
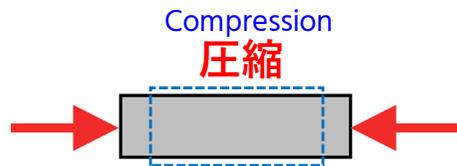


**We will focus on ISOTROPIC,  
HOMOGENEOUS and LINEAR ELASTIC  
materials**

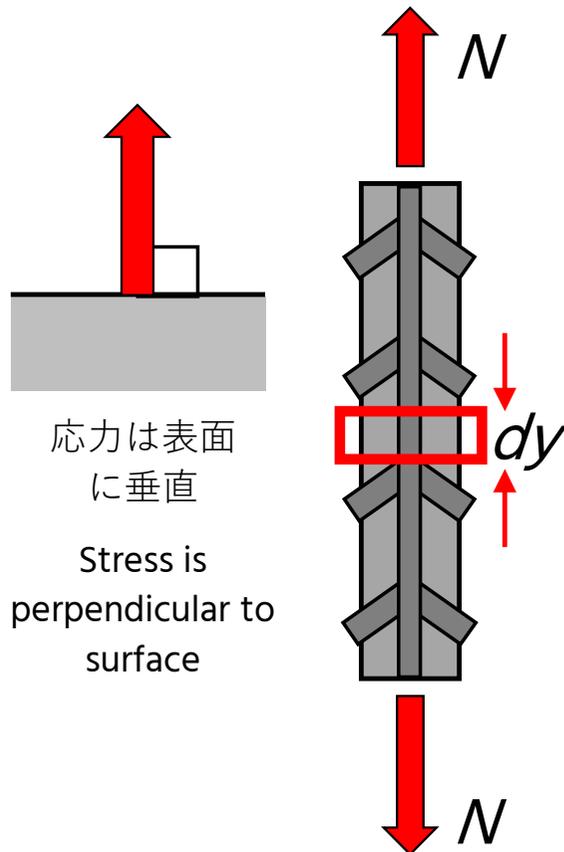
**均質等方性線型弾性材料  
に焦点を当てる**



# ひずみ・Strain



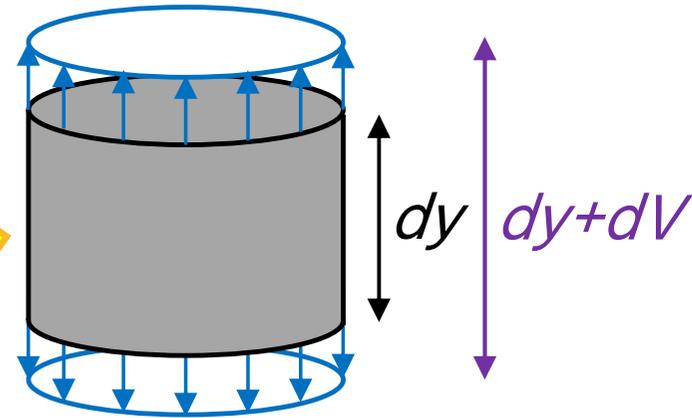
# 垂直ひずみ・Normal Strain



Elongation due to  
tensile load  $N$

引張荷重  $N$  により、  
鉄筋が伸びる

Change in length over distance  $dy$   
 $dV = dy$ における長さの変化

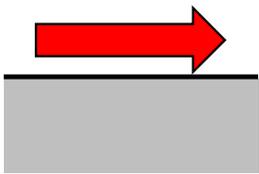


Normal strain is a measure of deformation per unit length

垂直ひずみ( $\epsilon$ ): 単位長さ当たりの変形

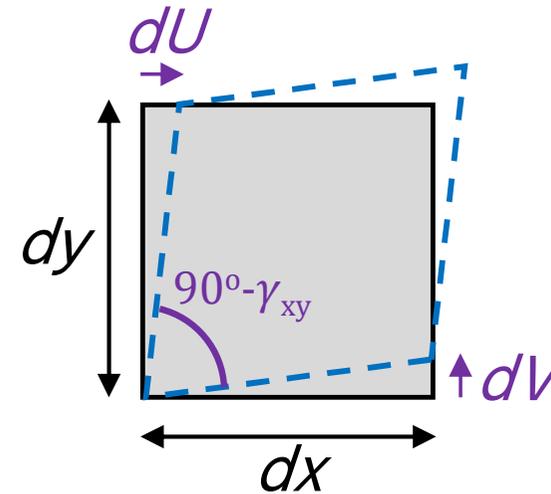
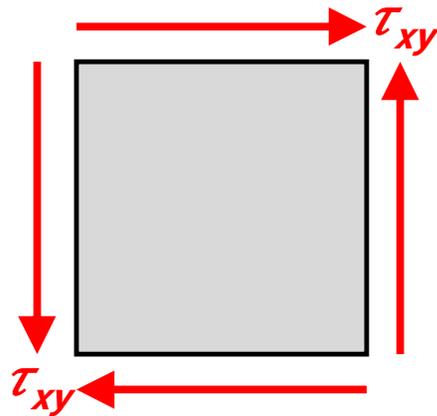
$$\epsilon_{yy} = \frac{dV}{dy}$$

# せん断ひずみ・Shear Strain



応力は表面  
に平行

Stress is parallel  
to surface

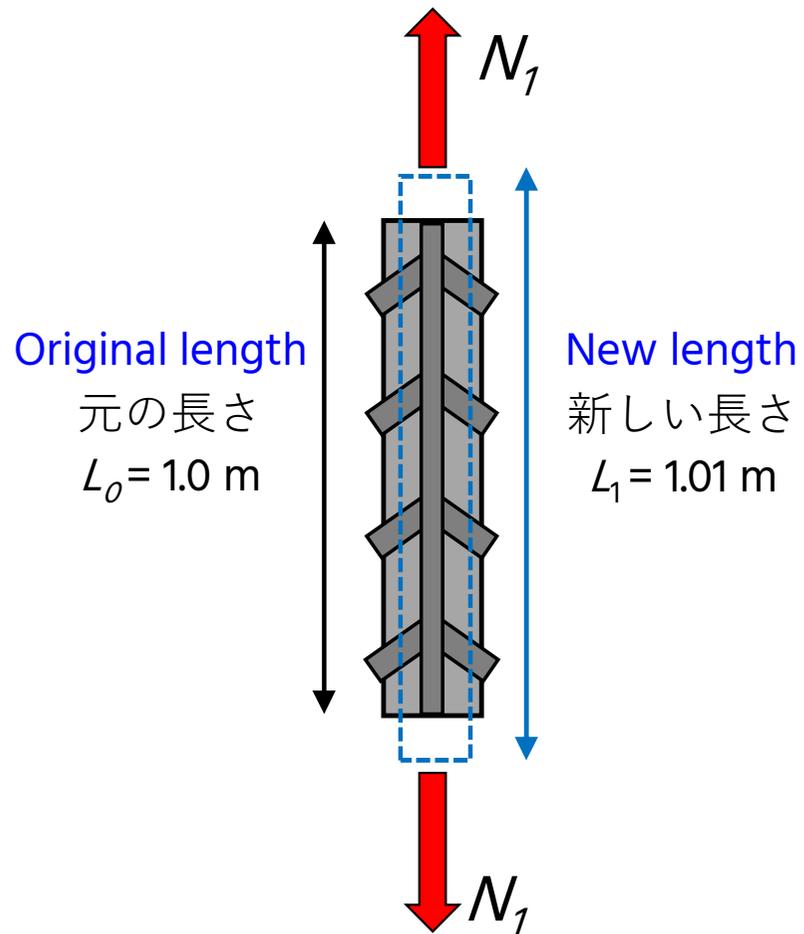


Shear strain is the change in angle from a right angle

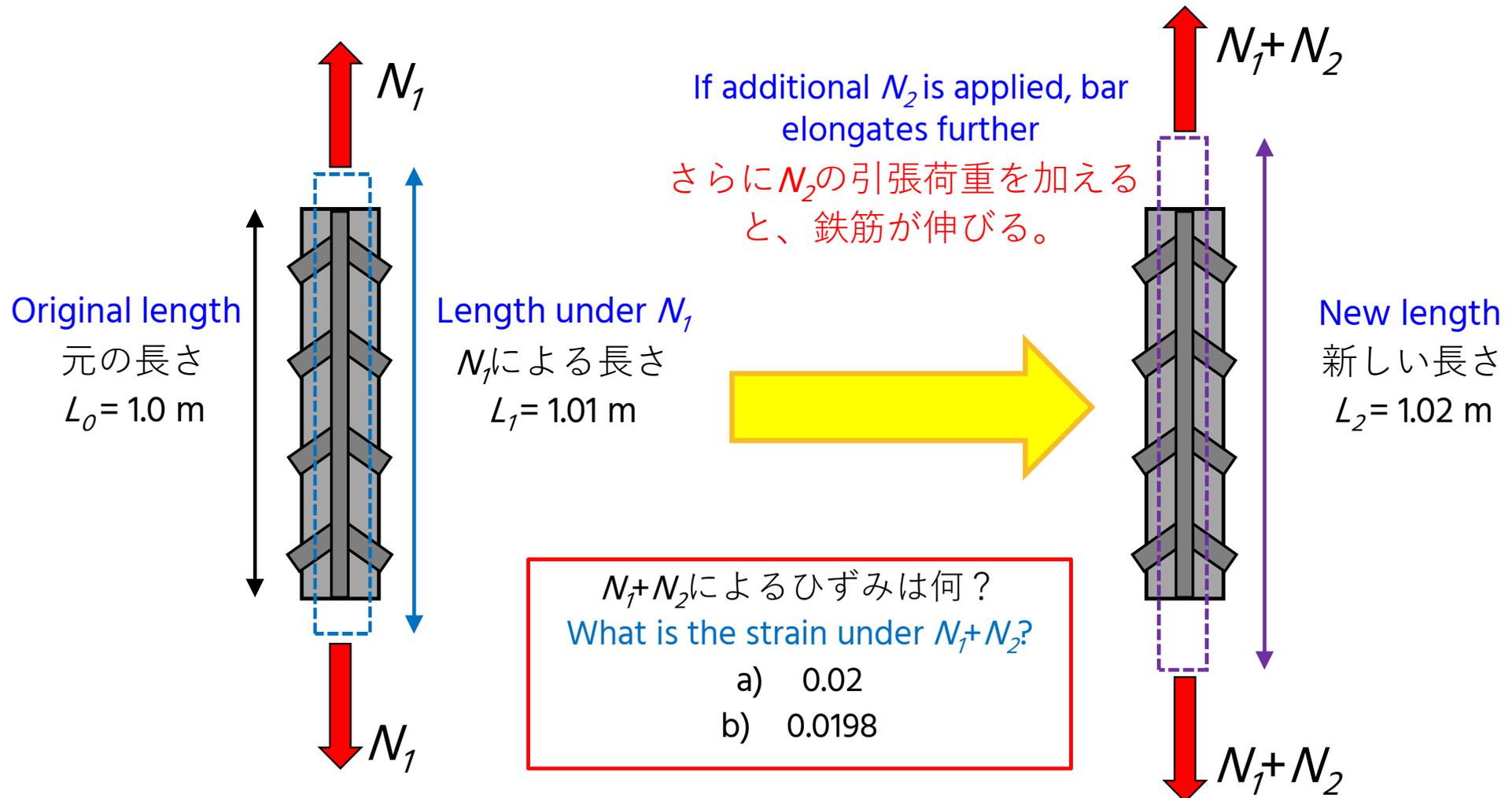
せん断ひずみ( $\gamma$ ): 直角からの変化量

$$\gamma_{xy} = \frac{dU}{dy} + \frac{dV}{dx}$$

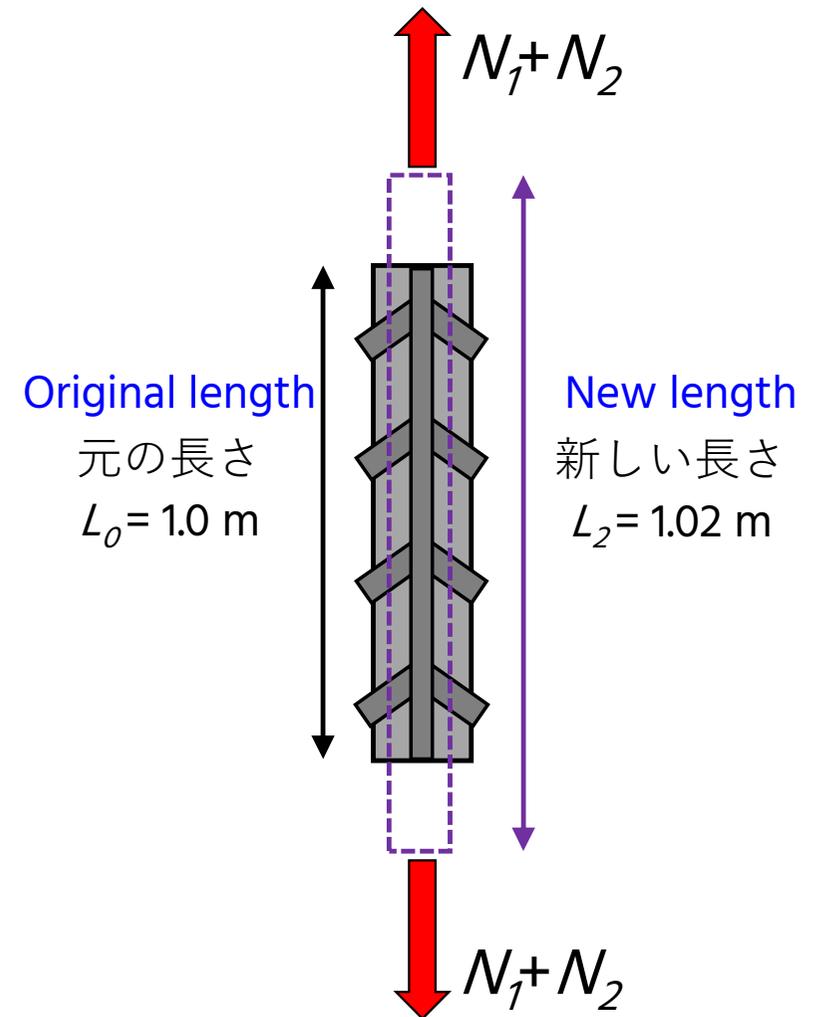
# 例) 引張荷重・Tension load example



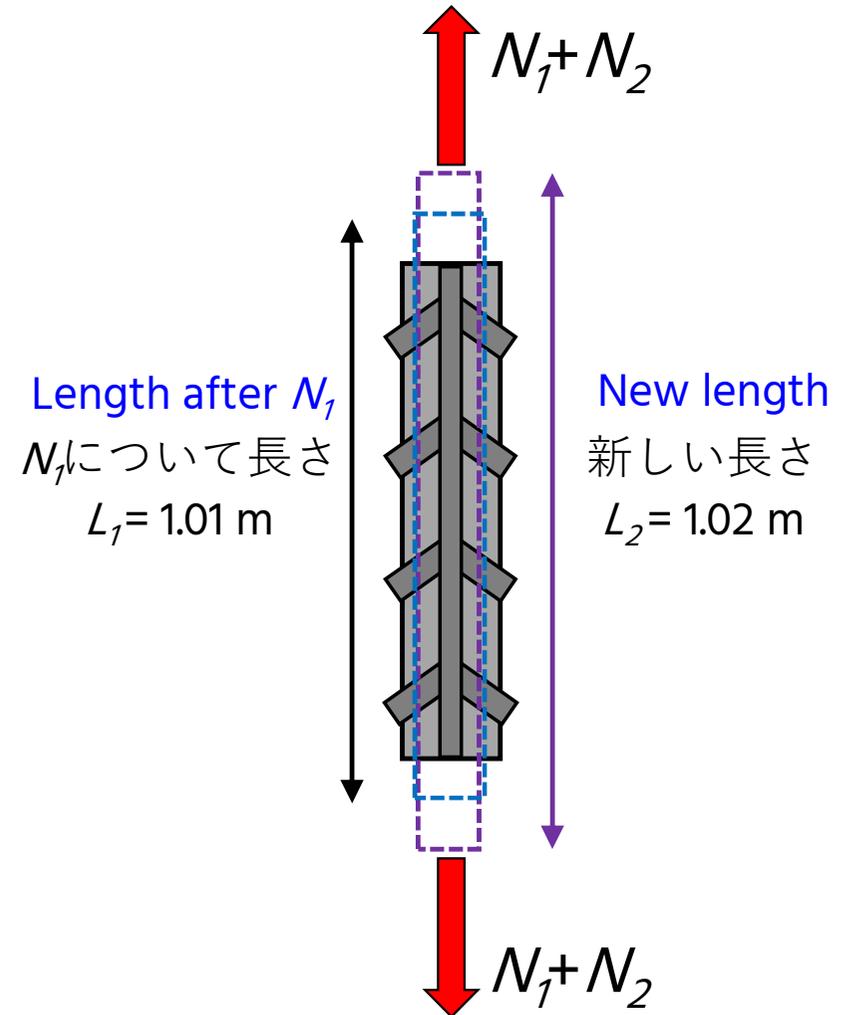
# 例) 引張荷重・Tension load example



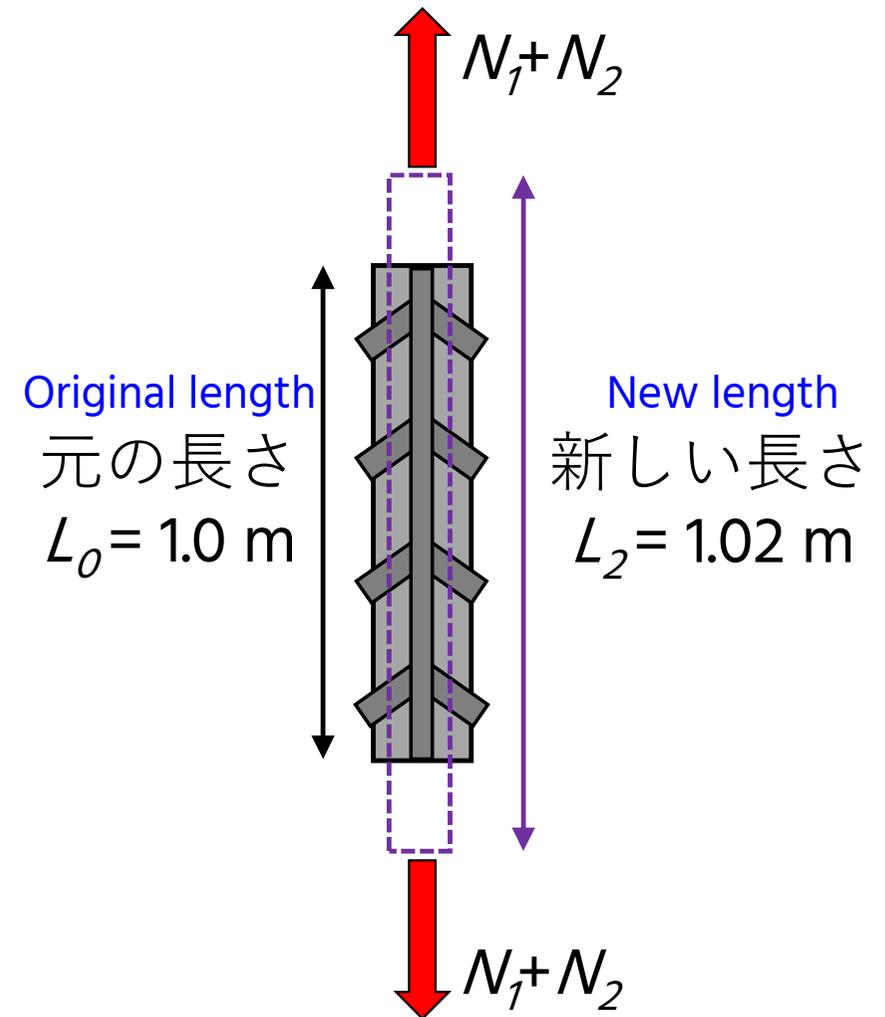
# 工学ひずみ・Engineering strain



# 真ひずみ・ True strain

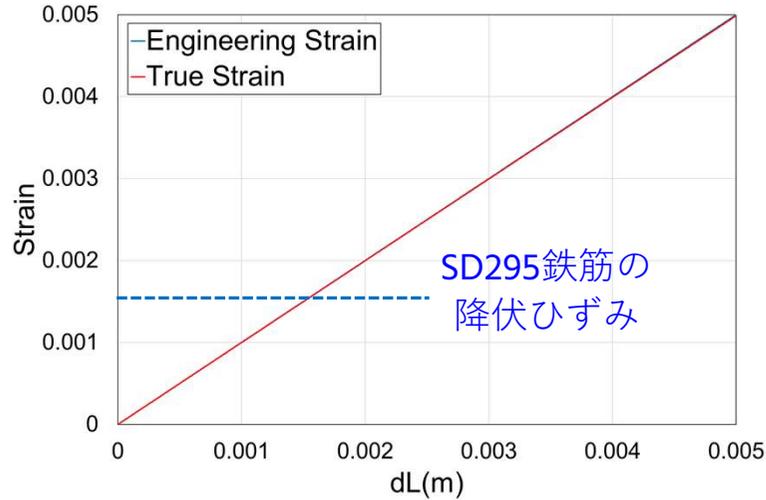


# 真ひずみ・True strain

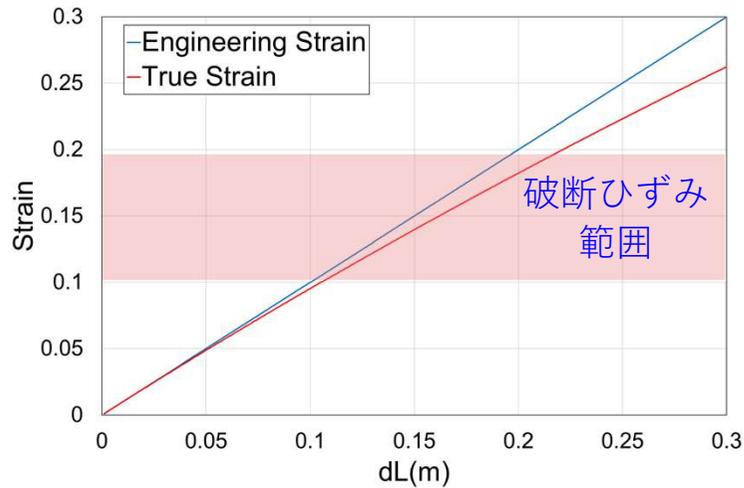


# 工学ひずみ・真ひずみ Engineering and true strain

低いひずみ



高いひずみ



Engineering Strain

工学ひずみ：

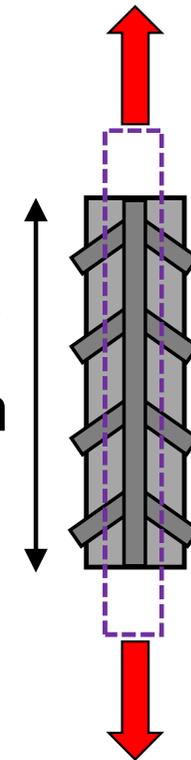
$$\frac{L - L_0}{L_0}$$

True Strain

真ひずみ：

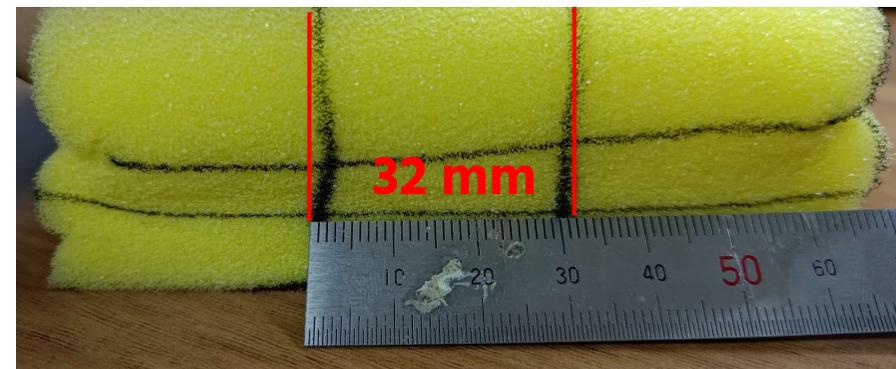
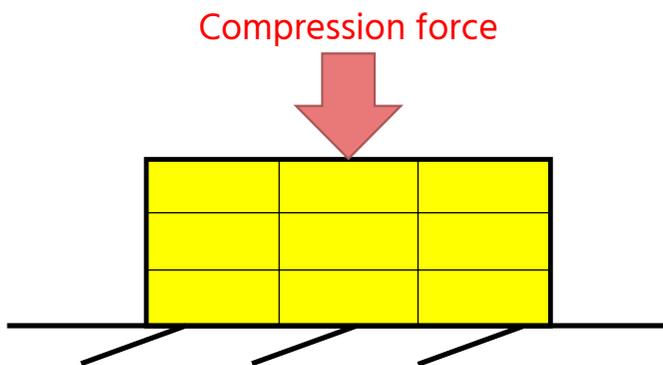
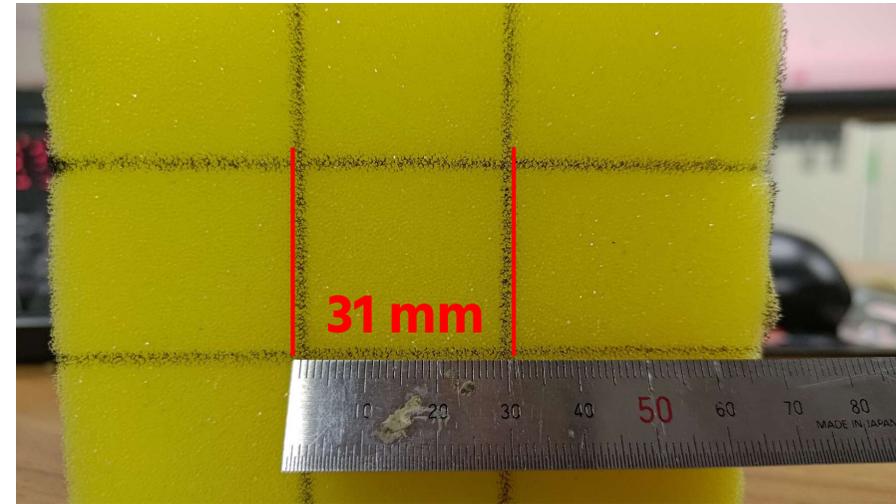
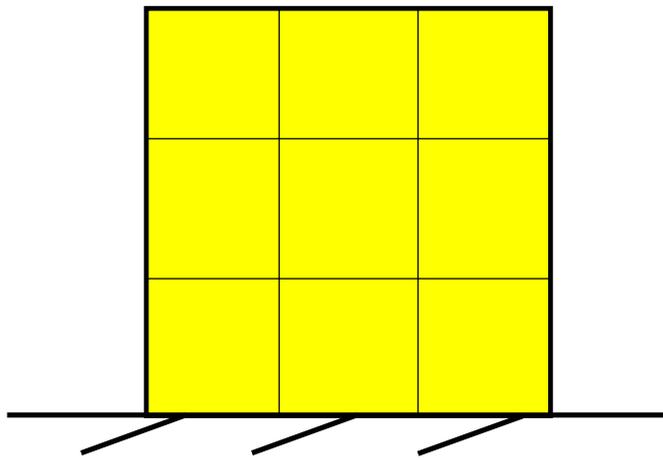
$$\ln\left(\frac{L}{L_0}\right)$$

元の長さ  
 $L_0 = 1.0 \text{ m}$

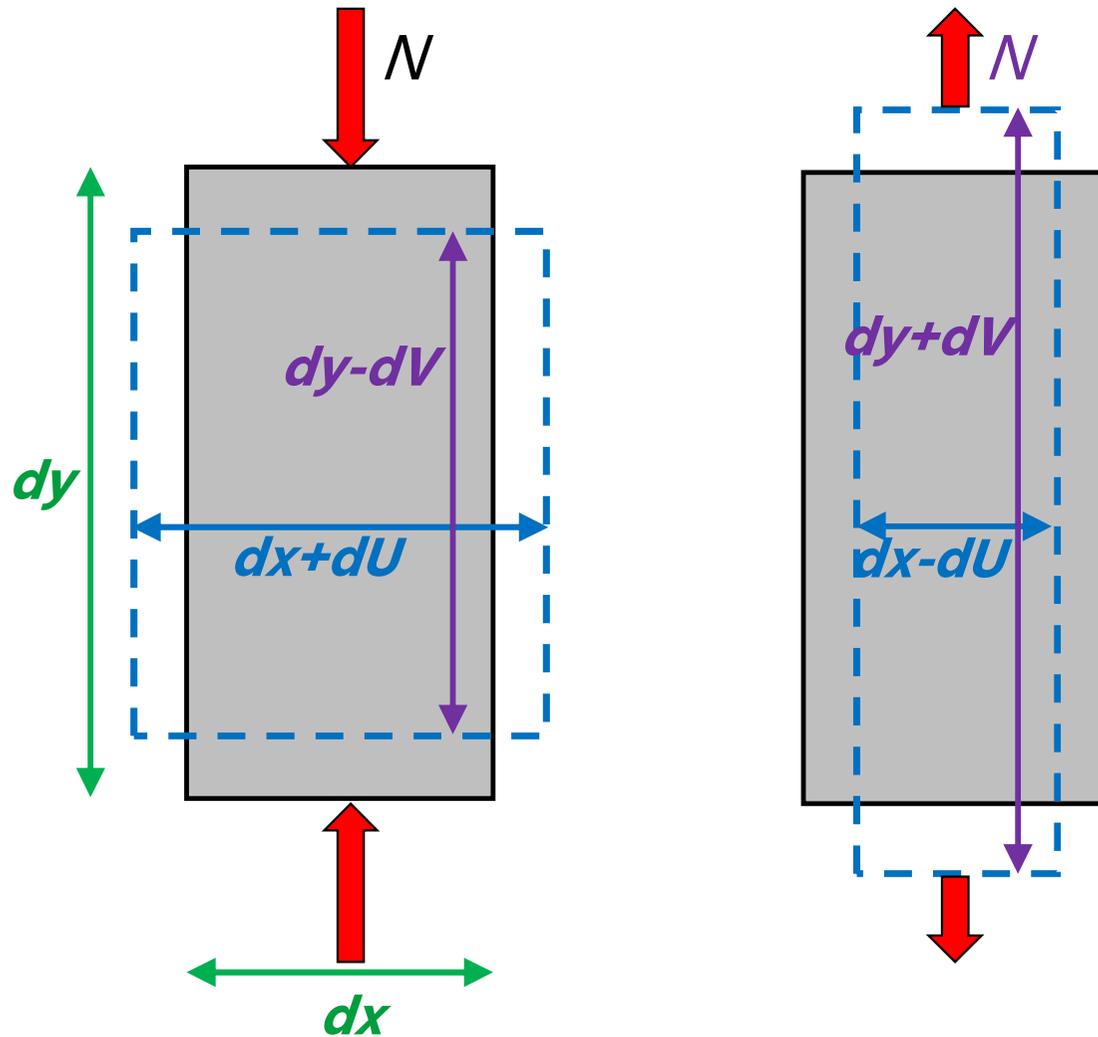


新しい長さ  
 $L = L_0 + dL$

# ポアソン比・Poisson's Ratio



# ポアソン比・Poisson's Ratio



Force  $N$  causes longitudinal strain ( $\varepsilon_y$ ) along  $y$  axis.

$N$ は  $y$  方向縦ひずみ( $\varepsilon_y$ )を引き起こす

$$\varepsilon_{yy} = \frac{dV}{dy}$$

Deformation will also occur in the sideways direction. (lateral strain of  $\varepsilon_x$ )

横方向に変形もある(横ひずみ  $\varepsilon_x$ )

$$\varepsilon_{xx} = \frac{dU}{dx}$$

The strains can be related via Poisson's Ratio,  $\nu$  (~0.16 for concrete, ~0.3 for steel, ~0 for sponge) for isotropic homogeneous linear elastic materials

均質等方生線型弾性材料の横・縦ひずみは、ポアソン比（コンクリートで~0.16、鋼材で~0.3、スポンジ~0）を介して関係づけられる。

$$\nu = -\frac{\varepsilon_{xx}}{\varepsilon_{yy}} \text{ (Uniaxial loading case)}$$

# ひずみゲージ・Strain Gauge

ひずみはひずみゲージを用いて測定することができる  
Strains can be measured using strain gauges

ひずみゲージが取り付けられている面が伸びると、ワイヤーの長さは長くなり、せん断断面積は小さくなる  
If surface that strain gauge is attached to elongates, the wire length increases and its cross sectional area decreases.

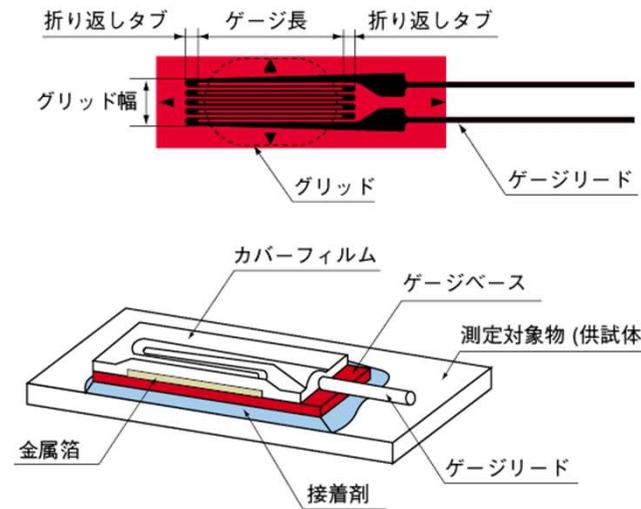
そのため、ひずみゲージの抵抗を増えている  
This causes the resistance of the gauge to increase

$$\varepsilon = \frac{1}{K} \frac{\Delta R}{R}$$

$\Delta R$  - 抵抗変化量・Change in resistance

$R$  - 最初の抵抗・Initial resistance value

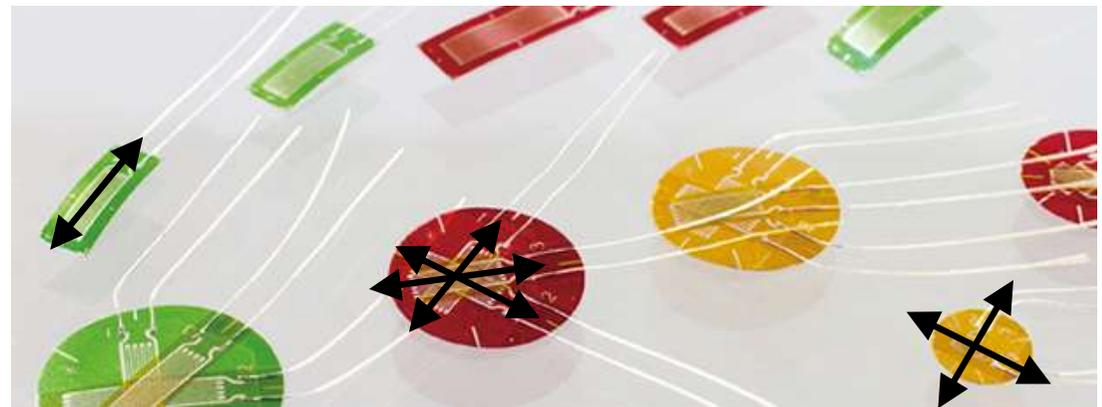
$K$  - ゲージ率(~ 2.0)・gauge factor (~ 2.0)



Gauge components<sup>1</sup>



Use in testing



Various types of strain gauges<sup>1</sup>

<sup>1</sup> 株式会社東京測器研究所・Tokyo Measuring Instruments Laboratory Co., Ltd  
[https://tml.jp/knowledge/strain\\_gauge/about.html](https://tml.jp/knowledge/strain_gauge/about.html)

# ひずみゲージ・Strain Gauge

抵抗の変化は、Wheatstone Bridge Circuitの入力電圧 ( $E$ ) と出力電圧 ( $e$ ) を比較することで測定できる：

Change in resistance can be measured by comparing the input voltage ( $E$ ) and output voltage ( $e$ ) of a Wheatstone Bridge Circuit:

$$e = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} E$$

This is NOT Young's Modulus!!!

$R_1$  - ひずみゲージ時の抵抗値・strain gauge resistance

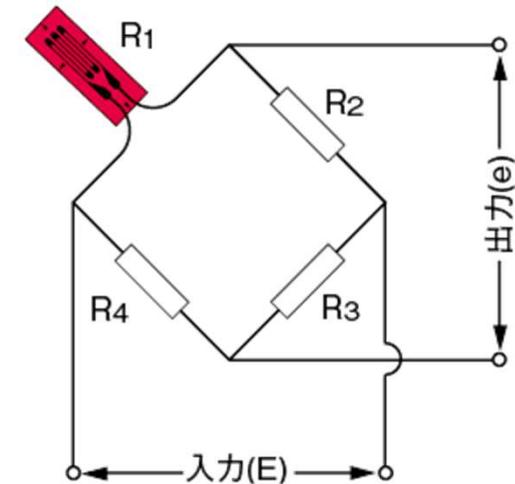
$R_2$ - $R_4$  - 固定抵抗の抵抗値・resistors resistance

$R_1$ ~ $R_4$ の初期抵抗値を $R$ とし、ひずみゲージの抵抗値変化が $\Delta R$ 発生したとする  
If the initial resistance of  $R_1$ ~ $R_4$  is  $R$ , and a change in resistance of the strain gauge of  $\Delta R$  occurs:

$$\Delta e = \frac{(R+\Delta R)R - R^2}{(2R+\Delta R)(2R)} E = \frac{\Delta R}{4R+2\Delta R} E \approx \frac{E \Delta R}{4 R} \approx \frac{E}{4} K \varepsilon \quad (R \gg \Delta R)$$

ひずみは出力電圧の変化によって決定することができる

Thus, strain can be determined by a change in output voltage



Wheatstone bridge Circuit<sup>1</sup>



<sup>1</sup> 株式会社東京測器研究所・Tokyo Measuring Instruments Laboratory Co., Ltd  
[https://tml.jp/knowledge/strain\\_gauge/about.html](https://tml.jp/knowledge/strain_gauge/about.html)